Additional Document: Cooperative Driving between Autonomous Vehicles and Human-driven Vehicles Considering Stochastic Human Input and System Delay

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Abstract— **This document presents additional information for the formulation of cooperative driving between autonomous vehicles (AVs) and human-driven vehicles (IHVs) considering stochasticity in human inputs and system delay from different sources presented in [1].**

HUMAN INPUT TRANSITION MODEL CONSTRUCTION

From the HMM, we get

$$\text{Emission probability:}\quad P(u_k^h|S_k,a_k) \tag{1}$$
$$\text{Transition probability:}\quad P(S_{k+1}|S_k,a_k). \tag{2}$$

Denoting by $\mathbb{S}$ the set of all possible states, we obtain $P(u_{k+1}^h|u_k^h,a_k)$ as

$$\begin{aligned} P(u_{k+1}^h|u_k^h,a_k) &= \sum_{S_k\in\mathbb{S}} P(u_{k+1}^h,S_k|u_k^h,a_k) \\ &= \sum_{S_k\in\mathbb{S}} P(u_{k+1}^h|S_k,u_k^h,a_k)P(S_k|u_k^h,a_k) \\ &\propto \sum_{S_k\in\mathbb{S}} P(u_{k+1}^h|S_k,u_k^h,a_k)\underbrace{P(u_k^h|S_k,a_k)P(S_k|a_k)}_{\text{Bayes' Rule}}. \end{aligned}$$

From (1)–(2), u_{k+1}^h depends on S_{k+1} and a_{k+1}, and S_{k+1} depends on S_k and a_k. Assuming a transition model $P(a_{k+1}|a_k)$, we conclude that u_{k+1}^h does not depend on u_k^h when conditioned on S_k and a_k. We then obtain

$$\begin{aligned} P(u_{k+1}^h|u_k^h,a_k) &\propto \sum_{S_k\in\mathbb{S}} P(u_{k+1}^h|S_k,a_k)P(u_k^h|S_k,a_k)P(S_k|a_k) \\ &\propto \sum_{S_k\in\mathbb{S}}\left(\sum_{S_{k+1}\in\mathbb{S}} P(u_{k+1}^h,S_{k+1}|S_k,a_k)\right)P(u_k^h|S_k,a_k)P(S_k|a_k) \\ &\propto \sum_{S_k\in\mathbb{S}}\left(\sum_{S_{k+1}\in\mathbb{S}} P(u_{k+1}^h|S_{k+1},S_k,a_k)P(S_{k+1}|S_k,a_k)\right) \\ &\qquad \times P(u_k^h|S_k,a_k)P(S_k|a_k), \end{aligned} \tag{3}$$

where $P(u_{k+1}^h|S_{k+1},S_k,a_k)$ is further written as

$$\begin{aligned} P(u_{k+1}^h|S_{k+1},S_k,a_k) &= \sum_{a_{k+1}} P(u_{k+1}^h,a_{k+1}|S_{k+1},S_k,a_k) \\ &= \sum_{a_{k+1}} P(u_{k+1}^h|S_{k+1},a_{k+1},S_k,a_k) \\ &\qquad \times P(a_{k+1}|S_{k+1},S_k,a_k) \\ &= \sum_{a_{k+1}} P(u_{k+1}^h|S_{k+1},a_{k+1})P(a_{k+1}|a_k). \end{aligned} \tag{4}$$

Thus, to compute $P(u_{k+1}^h|u_k^h,a_k)$ in (3), we need (1), (2), $P(S_k|a_k)$ and $P(a_{k+1}|a_k)$. While $P(S_k|a_k)$ and $P(a_{k+1}|a_k)$ can be learned from observations in experiments, in our simulations we assume that $P(S_k|a_k)$ is a uniform distribution and that there is no transition of a_k, i.e.,

$$P(a_{k+1}|a_k) = \begin{cases} 1 & a_{k+1} = a_k \\ 0 & a_{k+1} \neq a_k. \end{cases} \tag{5}$$

At each time step, a_k can be estimated as a distribution $P(a_k)$ by a monitoring system of the driver's actions. For our simulations, we consider a simplified probability model for $P(a_k)$ given in Table I. The driver's actions include speeding up (s^u), slowing down (s^d), and normally driving (s^c), and the current human action is $a_k \in \{s^d, s^c, s^u\}$ as explained in Section III.A of reference [1].

TABLE I
A SIMPLIFIED MODEL USED FOR $P(a_k)$

Conditions	$P(a_k = s^d)$	$P(a_k = s^c)$	$P(a_k = s^u)$
$u_k^h \leq -0.2$	0.9	0.05	0.05
$-0.2 < u_k^h < 0.2$	0.05	0.9	0.05
$u_k^h \geq 0.2$	0.05	0.05	0.9

Based on $P(a_k)$ and $P(u_{k+1}^h|u_k^h,a_k)$, we compute $P(u_{k+1}^h|u_k^h)$ as

$$\begin{aligned} P(u_{k+1}^h|u_k^h) &= \sum_{a_k\in\mathbb{A}} P(u_{k+1}^h|u_k^h,a_k)P(a_k|u_k^h) \\ &\propto \sum_{a_k\in\mathbb{A}} P(u_{k+1}^h|u_k^h,a_k)P(u_k^h|a_k)P(a_k) \\ &\propto \sum_{a_k\in\mathbb{A}} P(u_{k+1}^h|u_k^h,a_k)\left(\sum_{S_k\in\mathbb{S}} P(u_k^h|S_k,a_k)P(S_k|a_k)\right) \\ &\qquad \times P(a_k). \end{aligned} \tag{6}$$

This transition model $P(u_{k+1}^h|u_k^h)$ is used as the cHMM model in the paper [1].

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