Digital Control Systems
MAE/ECEN 5473

Pulse Transfer Function (PTF)

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August 14, 2023

PTF in different configurations: Series I

Series II: two blocks in series

Series III: blocks with digital control in series

Digital control is PID

Closed-loop system

General procedure

1. Assign a variable before each sampler. If the output has no sampler, put a ficticious one.
2. The output of each sampler is the starred variable.
3. Express the sampler inputs & system output in terms of the sampler outputs and system input.
4. Take the star transform & solve for the system output in terms of the input.
5. Convert to Z-domain from the star transforms.

Example

Example (Radar tracking)

Stability of TF in CT

$G(s) = \frac{p(s)}{q(s)}$, $p(s)$, $q(s)$ are polynomials of s.

When is $G(s)$ stable?

The stability of $G(s)$ depends *only* on its poles (i.e., the roots of $q(s) = 0$).

- Stable
- Marginally stable
- Unstable

Example

Stability of PTF in DT

$$G(z) = \frac{p(z)}{q(z)},\ p(z),\ q(z) \text{ are polynomials of } z.$$

- $z = e^{Ts} =$
- In CT, stability is represented by
- Then in DT, stability region is given by

Stability of $G(z)$

- Stable
- Marginally stable
- Unstable

Example

Determine stability of $G(z)$

- MATLAB: poles(), pzmap(),
- Compute the poles
- Routh Hurwitz table (from CT): $q(z)\big|_{z=\frac{w+1}{w+1}}$
- Jury test

Jury table

denominator of $G(z)$: $q(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$.

0.
1.
2.
3.

4. Create a Jury table

Jury test

If Condition 1-4 from previous page are satisfied, $G(z)$ is stable. Otherwise it is unstable.

Examples

$q(z) = -z^3 + z^2 + 0.5z + 1$

$q(z) = z^3 + 0.5z^2 - 1.34z + 0.24$

Example: determine the range of K

Sampling theorem: Motivation example

Consider a sinusoid $A \sin(wt + \phi)$. Assume w is known. How many samples within one period are needed to reconstruct the sinusoid (i.e., estimate A and ϕ).

Sampling theorem

Define $\omega_s = \frac{2\pi}{T}$, where T is the sampling time. Assume that the highest frequency component of a CT signal $x(t)$ is ω_1 rad/s. Then if $\omega_s > 2\omega_1$, $x(t)$ can be reconstructed from the sample signal $x^*(t)$. ideally!!

In practice

$\omega_s \sim (10 - 30)\omega_1$.

Why?

Aliasing $\omega_s < 2\omega_1$

Use ZOH to reconstruct $x(t)$