Welcome

Digital Control Systems
MAE/ECEN 5473

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Course logistics

- Syllabus available online `my.okstate.edu`
- Class meetings: Monday, Wednesday 8 - 9:15 am,
  online/asynchronous video recording
  (Classroom Building 322 reserved for discussions and exams).
- Prerequisites: MAE 4053 (Automatic Control Systems) or ECEN 4413. Familiarity with MATLAB.
- Textbook: K. Ogata, *Discrete-Time Control Systems*, Prentice-Hall, Second edition, 1995 (ISBN 0-13-034281-5).

Course logistics

- Grading: Homework -20%, Project - 20%, Quizzes - 5%, Exams (5%, 25%, 25%).
- Two-week advance notice will be given for the mid-term exam. The final exam will be in the finals week.
- Office hour: online by appointment
- Contact info: `he.bai@okstate.edu`
- Academy integrity: `academicintegrity.okstate.edu`
- Current syllabus attachment: `https://academicaffairs.okstate.edu/student-support/index.html`
- Class notes/videos will be available online.

Ground rules

- No TA: Use office hours and emails
- Attendance
- Minimize cell phone usage (silent or airplane mode)
- Late HWs will be penalized (n days late $= n \times 10\%$ off).
- Discussions and questions are always welcome.
- Exams: take-home/in-person (close book with one cheat sheet; basic calculator)

Online teaching: Two modes

- Microsoft Teams used for live streaming
- Asynchronous video recordings
- Starting in the 3rd week, weekly online discussions (30 min): indicate your availability
  `https://www.when2meet.com/?20810393-QAsXZ`

Introduction: About you and your academic interests

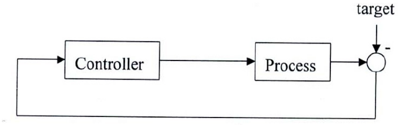
Introduction: Control systems

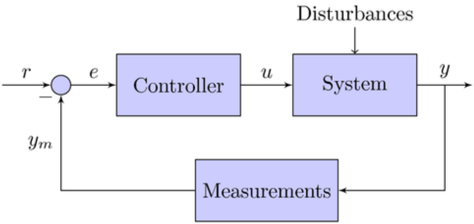
- Control systems: using controllers (sensors and actuators) to regulate behaviors of systems.
  - Regulate output of a system
- Open-loop control
  - Examples: washer, sprinkler system, pushing a box
  - No feedback from system outputs.

Feedback control is needed.

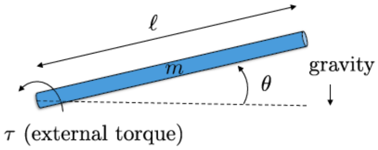
Feedback control systems

- Objective is to drive the output of the process to the target
- Disturbances (to the system and controller) and noise (to the sensor) can come into the system





Example



Equation of motion (with damping $-b\dot{\theta}$): Can you derive it?

$$\frac{m\ell^2}{3}\ddot{\theta} + mg\frac{\ell}{2}\cos\theta = \tau - b\dot{\theta}$$

- Objective is to regulate θ to θ_r
- Different flavors of control designs: transfer function (classical), state space (modern), adaptive control, optimal control, robust control, digital control, $\cdots$

Digital control systems

Using computer or digital controllers to in control systems

- Availability of low-cost digital computers
- Advantage of using digital signals: reduced cost (control multiple loops using the same computer); flexibility in response to design changes (only need to update software) and (3) noise immunity (less sensitive to noise than analog signals)
- Applications in engineering, finance, social systems, include robotics, chemical process, power plant, car engine, cash reserve, population,...

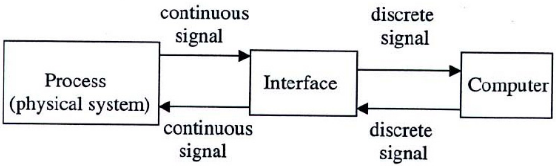
Examples:

- X-29 forward-swept airplane design - reference G. Kaplan, "The X-29: Is it coming or going?", IEEE Spectrum 1985. Then nowadays UAV autopilot design based on sensor data coming at a certain update rate (100 Hz IMU, 1 Hz GPS)

X-29

- Designed to sacrifice stability for high maneuverability and speed (forward-swept wings: reduce drag and increase maneuverability)
- Has to be controlled by computer (digital and analog backup, update rate 40 Hz)
- Computer control: due to lesser stability, increase the gain and bandwidth of components in flight to twice and three times their normal values (more sensitive to noise). Particular for pitch control: used a filter to estimate pitch acceleration (complementary filter) rather than measuring it directly.

A diagram of DCS



Interface: Sampling and hold (S/H), A/D, D/A converters

- Sampling: replace continuous time signals by a sequence of values at discrete time
- Hold: hold the sampled sensor value for a certain amount of time (because the A/D converter converts the voltage to a digital number via a digital counter, which takes time to reach the correct digital number.)
- A/D (encoder): converts an analog signal to a digital signal (quantization involved)
- D/A (decoder): converts a digital signal to an analog signal

Different kinds of signals (Fig. 1-1, page 2)

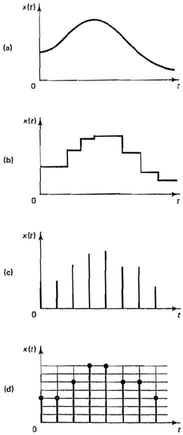


Figure 1–1 (a) Continuous-time analog signal; (b) continuous-time quantized signal; (c) sampled-data signal; (d) digital signal

- Continuous time analog signal (a), Sampled-data signal (c), digital signal (d)

Additional signals:

- S/H signal (after holding becomes similar to (b))
- Continuous time quantized signal (b)

Quantization error

- Amplitude quantization: analog signal by a finite number of discrete states
- Binary system of n bits: represent 2^n amplitude levels
- Quantization level (Q): Full scale range (FSR)$/2^n$, or least significant bit
- Quantization error: between 0 and $1/2Q$, i.e., $0 \leq |x - x_q| \leq 1/2Q$.
- Increasing n reduces quantization error
- Approximate quantization error as uniformly distributed random noise in $(-1/2Q, 1/2Q)$: variance of the noise is $Q^2/12$.

Computer: Control designs

- Most of modern control laws are implemented in discrete time (due to the use of computers).
- Design continuous controls and implement them in discrete time (is this enough?)
- Assume high update rates. Also sometimes the system is in discrete time only (e.g., identified through experiments)
- Directly design a discrete time control based on a discrete time system
  - Need to characterize the total effect of *the process and the interface* to design such a controller

Process: Linear time-invariant system

Definition

A linear system: principle of superposition applies. Consider the input/output pairs (u_1, y_1) and (u_2, y_2) of a system. The system is linear if $(\alpha u_1 + \beta u_2, \alpha y_1 + \beta y_2)$ is also an input/output pair.

Linear systems may be described by differential equations or difference equations.

Definition

An LTI system: coefficients of DEs/ODEs are time-invariant (system properties do not change w.r.t. time).

$$\ddot{x} + k\dot{x} + gx = u$$

$$\ddot{x} + k\dot{x} + (g + \epsilon \cos \omega t) = u$$

$$x(k+1) = ax(k) + bu(k), \quad y(k) = x(k)$$

$$x(k+1) = a^k x(k) + bu(k), \quad y(k) = x(k)$$

Review of continuous time control for LTI

Example

The single-link robot example: second order system

$$\frac{m\ell^2}{3}\ddot{\theta} + mg\frac{\ell}{2}\cos\theta = \tau - b\dot{\theta}$$

$$\tau = mg\frac{\ell}{2}\cos\theta + u$$

$$\Rightarrow \frac{m\ell^2}{3}\ddot{\theta} + b\dot{\theta} = u$$

Design $u(t)$ such that $\theta(t)$ goes to a specific location, say, θ_c as $t \to \infty$ (asymptotic stabilization).

Control design and analysis methods:

- Transfer function (Laplace transform)
- State space

Transfer function

Laplace transform

$$F(s) = \int_0^{\infty} \exp(-st) f(t) dt$$

Let $f(s)$ be the Laplace transforms of $f(t)$.

Example

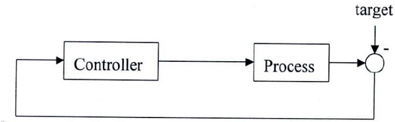
$f(t) = 1 \Rightarrow F(s) = \frac{1}{s}$
$\dot{f}(t) \Rightarrow sF(s) - f(0)$ What about $\ddot{f}(t)$?

Previous example gives the following transfer function

$$\frac{m\ell^2}{3}(s^2\Theta(s) - s\theta(0) - \dot{\theta}(0)) + b(s\Theta(s) - \theta(0)) = U(s).$$

Ignoring initial conditions: Transfer function $P(s) = \frac{\Theta(s)}{U(s)} = ?$

Traditional analysis and design methods



Process: $P(s)$, Control: $C(s)$

Analysis: Given a feedback controller $C(s)$, analyze the stability and the performance of the system shown below.

- Closed-loop transfer function
- Analysis tools – Routh hurwitz: stability of closed-loop $H(s)$, root locus: Stability of $H(s)$ w.r.t. gains, Bode plot: gain and phase margin, robustness to gains and delays.

Design: Construct $C(s)$ to satisfy certain system performance (e.g., settling time, steady state error, overshoot, rising time, ...)

- Design methods: Root locus based designs, Phase lead/phase lag controllers, PID, ...

State space methods

$$\frac{m\ell^2}{3}\ddot{\theta} + b\dot{\theta} = u$$

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$. Then

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} ? \\ ? \end{bmatrix} u(t), \quad y = [1 \;\; 0] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Analysis: Assume we are given a closed-loop system (e.g., $u = k_1 y$).

- Stability analysis: eigenvalues of system matrix, Lyapunov function

Design: How to design a feedback controller $u(t)$ (i.e., making use of the output information)

- Pole placement, observer design, LQR

This course

Developing similar tools for digital control systems

- Analysis (traditional and state space methods)
  - Z-transform, difference equations
  - Effects of sampling, zero-order hold, ...
  - Transfer function and stability
  - State space analysis (Stability, controllability/observability)
- Designs
  - Traditional approaches: root-locus and frequency methods.
  - State space approaches: pole placement, observer design, state estimation.
  - Optimal control approaches: linear quadratic optimal control.

Next class (Aug. 28)

- DCS and Z-transform
- Read Chapter 1 and Chapter 2-1,2
- **Entry exam** (released) to recap some important concepts in continuous-time control systems
- Wednesday (Aug. 23) class is canceled to give your time to complete the entry exam