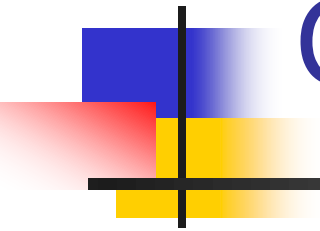
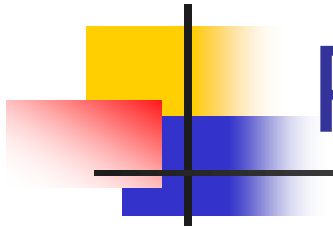


Computer Methods (MAE 3403)

Determine polynomial via systems of linear equations



Polynomial finding

- Determine the coefficients of the polynomial

$$y = a_0 + a_1*x + a_2*x^2 + a_3*x^3$$

  that passes through the points (0, 10), (1, 35), (3, 31), and (4, 2).

- Formulate it as a system of linear equations
  - The solution corresponds to the coefficients a_0, ..., a_3

Linear equations

- Passing through the point (0, 10)

  $10 = a_0 + a_1 * 0 + a_2 * 0^2 + a_3 * 0^3$

- Passing through the point (1, 35)

  $35 = a_0 + a_1 * 1 + a_2 * 1^2 + a_3 * 1^3$

- Passing through (3, 31), and (4, 2)

  $31 = a_0 + a_1 * 3 + a_2 * 3^2 + a_3 * 3^3$

  $2 = a_0 + a_1 * 4 + a_2 * 4^2 + a_3 * 4^3$

4 equations 4 unknowns

- In the Ax = b form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 35 \\ 31 \\ 2 \end{bmatrix}$$

$$A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}, \; x=\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}, \; b= \begin{bmatrix} 10 \\ 35 \\ 31 \\ 2 \end{bmatrix}$$

- Solve it using Gauss Elimination

Can we automate the process of creating A and b

- Given the order of the polynomial, n, and given a list of passing points $P = [[x_1, y_1], [x_2, y_2], \ldots, [x_m, y_m]]$
  - Assume $m = n + 1$

Figure out m from P
Initialize A and b with the right dimensions
for j in range(0, m): # for the jth row of A and b
    Pull out the jth point as P_j from P
    Assign the corresponding x_j and y_j from P_j
    Assign the jth row of b: $b[j][0] = y_j$
    Assign the jth row of A: $A[j] = [1\ x_j\ x_j^2\ \ldots\ x_j^n]$ (How?)

Online quiz problem

- Determine the fourth-degree polynomial y(x) that passes through the points (0, −1), (1, 1), (3, 3), (5, 2), and (6, −2).

$$y = a_0 + a_1*x + a_2*x^2 + a_3*x^3 + a_4*x^4$$

  - What are the dimensions of A and b?
  - What are the coefficients of a_i?